# INHOMOGENEOUS OPEN-ENDED RESONATORS AS MICROWAVE SENSOR ELEMENTS

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## SUMMARY

Inhomogeneous open-ended coaxial line resonators terminated by multilayer media have been investigated with regard to their application for the determination of the (local) permittivity of dielectric materials. A multisectional transmission-line model is presented which characterizes the experimental arrangement. The fringing capacitance at the open aperture is computed for the electrostatic case. Experimental and theoretical results for the attainable frequency shift due to the variation of the permittivity of the termination structure, compared with the respective case of the homogeneous resonator, will be presented.

## INTRODUCTION

By using open-ended coaxial lines the permittivity of biological substances and dielectric materials - especially its local or spatial distribution - is determined by reflection-coefficient measurement techniques or by open cavity measurement methods. Measurements can be made over a wide frequency range and with small aperture dimensions if the geometrical dimensions and the dielectric material, filling the coaxial line, are selected appropriately so that only the TEM-wave can propagate. Under these conditions higher order modes decay rapidly and radiation losses can be neglected.

Several authors have been engaged in the problem of permittivity determination using open-ended coaxial lines (e.g. /1-6/). A survey is given in /7/. Solutions of the scattering at the aperture of a coaxial line in contact with a dielectric sample which is infinitely extended are presented for the dynamic case in /2/ and for the static case in /1,5,6/. Additionally in /5/ the open-ended coaxial line is terminated by a two-layer medium.

Inhomogeneous coaxial lines are well known. For example they are applicated as transmission-line transformers and recently as closed cavity elements (/8,9/). Additionally in /9/ the Q-factor of a stepped resonant cavity is computed.

## THE THEORETICAL BACKGROUND

In this paper a coaxial probe is described, consisting of an open-ended coaxial line tuned to resonance, geometrically and dielectrically inhomogeneous in the direction of the mode propagation (Fig.1), i.e. in general with different inner

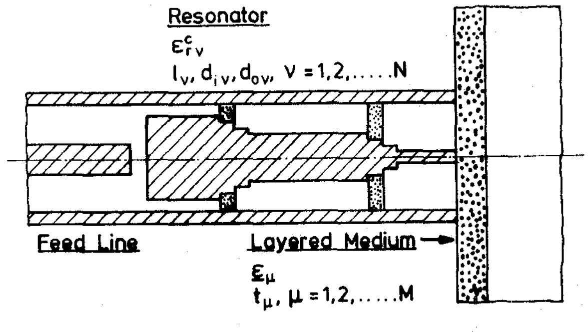


Fig.1: The inhomogeneous coaxial line resonator.

conductor diameters $d_{i\nu}$ ($\nu$ =1,2,3,...,N), different inner diameters of the outer conductor $d_{o\nu}$ and different geometrical lengths $l_\nu$, filled with dielectric materials of permittivity $\varepsilon^c_{r\nu}$, terminated by a layered medium consisting of dielectric sheet materials of thickness $t_\mu$ ($\mu$ =1,2,3,...,M) and permittivities $\varepsilon_{r\mu}$. The dielectric materials are assumed to be nonmagnetic, homogeneous, isotropic, linear and lossless. The conductivity of the metallic materials is assumed to be infinite.

In the lossless case the resulting equivalent circuit forms a multisectional Lecher-system tuned to resonance at zero input admittance or zero input impedance respectively /11/. In Fig.2 $\underline{Y}_N$ represents the imaginary input admittance. If the input is loaded by an inductive or a capacitive coupled line, the resonant condition has to be modified. Fig.3 shows the circuit arrangement of the $\nu$ -th single line section. It is characterized by a transmission line of the length $l_\nu$, the characteristic impedance $Z_{L\nu}$ and the phase coefficient $\beta_\nu$. The capacitance $C_\nu$ is introduced to describe the influence of the coaxial line discontinuity between the $\nu$ -th and the $\nu$ +1-th line section /10/.



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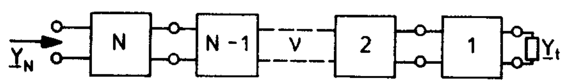


Fig.2: Equivalent circuit of the resonator.

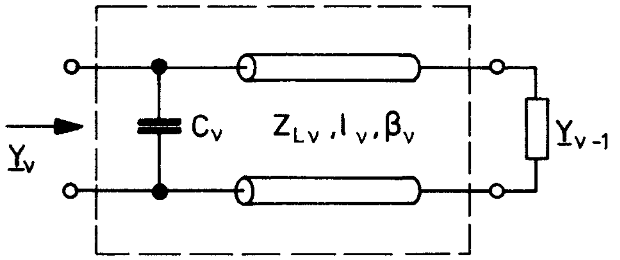


Fig.3: Equivalent circuit of one single section.

The input admittance $\underline{Y}_N$ of the total system including the fringing capacitance at the end of the line can be determined by means of the product of the ABCD-matrices of the single elements /11/. Alternatively it can be calculated by means of the repeated transformation of the terminating admittance $\underline{Y}_t = j\omega C_t$ over the line sections.

With respect to the determination of the local permittivity by open cavity measurement methods it is important to note that the capacitance $C_t$ is determined by the aperture dimensions $d_{il}$ and $d_{ol}$ and the dielectric material (permittivity $\varepsilon^c_{rl}$) filling the front part of the coaxial line. It is assumed that the length $l_1$ has been choosen so that the additional field disturbances due to the step discontinuities have been decayed at the aperture plane.

It is necessary to obtain a compromise between the requirements of small measured material volumes and the attainable sensitivity of the measurement set up. The sensitivity is defined by the ratio $\Delta f / \Delta \varepsilon_r$ where $\Delta f$ is the frequency shift yielded by a variation of the permittivity $\varepsilon_{r1}$ while all the other parameters are kept constant. If the requirement of a local measurement shall not be changed, i.e. that the effective measured material volume shall kept constant, an increased sensitivity can be realized using inhomogeneous probes. On the other hand the effective measured volume can be reduced if the sensitivity remains constant.

In Fig.4 this procedure of sensitivity increasing using inhomogeneous coaxial probes is illustrated for a two sectional line with an impedance step in plane A (see insert of Fig.4). Line section 1 is terminated by an open end. Line section 2 is assumed to be infinitely extended. The open circuit at section 1 is transformed into plane A. The result of this transformation may be interpreted as a lengthening of line section 2 according to the following equation:

$$\cot\Theta_2 = \frac{Z_{L1}}{Z_{L2}} \cot\Theta_1 , \qquad (1)$$

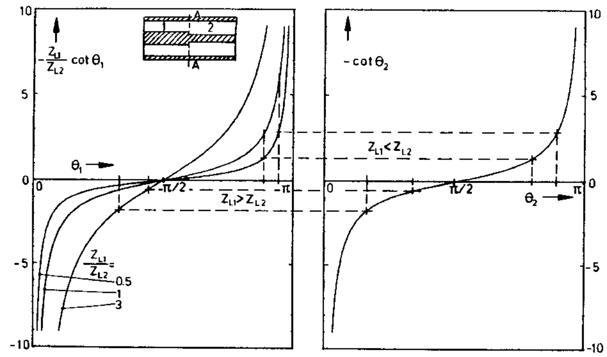


Fig.4: Impedance transformation procedure.

with $\Theta_1 = \beta_1 l_1$ ,

$\Theta_2 = \beta_2 l_2'$ and

$\beta_1$, $\beta_2$, $Z_{L1}$, $Z_{L2}$ phase coefficient and characteristic impedance of the line sections,

$l_1$ electrical length of line 1,

$l_2'$ hypothetical prolongation of line 2 in the step plane.

In Fig.4 the procedure according to equ.(1) is illustrated for two values of the impedance ratio $Z_{L1}/Z_{L2}=3$ and 0.5. The phase shift $\Theta_1$ has been varied (corresponding to a variation of the capacitance $C_t$) and the resulting variation of $\Theta_2$ has been evaluated graphically. For both impedance ratios the result is: $\Theta_2 > \Theta_1$ if the line length $l_1$ is choosen appropriately; this means that the phase shift which is caused by varying the length of line section 1 (or adequately by a capacitive loading of the open line) has increased compared to the case of the homogeneous line with the parameters $Z_{L1}$, $l_1$ and $\beta_1$. The impedance ratios, the position of the impedance steps and the value of the phase coefficients of the different line sections are decisive for these properties.

## EXPERIMENTAL RESULTS

Several possible configurations of stepped and tapered capacitively and inductively coupled resonators will be presented in the forum session.

In order to obtain experimental as well as theoretical results several multisectional stepped and tapered coaxial line resonators filled with air have been built. They are coupled to a 50 ohm feed line of inner diameter 1.6 mm, outer diameter 5 mm, a permittivity of the dielectric filling 2.1 (teflon). The sensitivity of the resonators has been tested with dielectric sheet materials of thicknesses $t_1$=1.27 mm and .6 mm (Fig. 1) and of the permittivity in the range $6 < \varepsilon_{r1} < 30$ while $t_2$ was taken very large and assumed to be infinite. The permittivity of the dielectric layer 2 was approximately equal 1 (polyfoam material).

In Fig.5 theoretical and experimental results of the resonant frequency as a function of the nominal permittivity (as given by the manufacturer) are presented. Two different 7-sectional inhomogeneous resonators (see Fig.1) with constant impedance ratios at each step are compared with the respective case of a homogeneous resonator. In each case the electrical lengths of the resonators were equal to lambda. The inner conductors were supported by teflon disks and the inner diameters of the outer conductors were constant $d_o=d_{o\nu}=5.0$ mm ($\nu=1,...,7$). The diameters of the

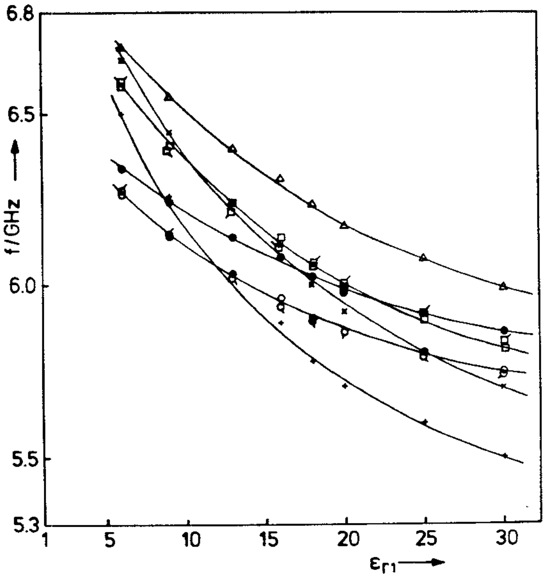


Fig.5: Frequency shift as a function of the permittivity $_{r1}$.
Homogeneous resonator
Experimental results: Theoretical results:
o $t_1$=1.27 mm    $t_1$=1.27 mm σ
• $t_1$=0.6 mm
Inhomogeneous resonator No.1
Experimental results: Theoretical results:
□ $t_1$=1.27 mm    $t_1$=1.27 mm □
Δ $t_1$=0.6 mm
Inhomogeneous resonator No.2
Experimental results:
+ $t_1$=1.27 mm
× $t_1$=0.6 mm

inner conductors were $d_i=d_{i1}$= 1.6 mm for the homogeneous resonator and the first section of the inhomogeneous resonators. The total electrical length of all resonators used was 41 mm.

The geometrical lengths of each section of the stepped resonators were ($\nu$ given in brackets): 7.75 mm (1), 1.0 mm (2), 1.0 mm (3), 19.5 mm (4), 1.0 mm (5), 1.0 mm (6), and 9.75 mm (7). The two stepped resonators differ from each other by their impedance ratios $Z_{L\nu}/Z_{L\nu+1}$. The line impedance of section 1 is $Z_{L1}$=68.37 ohms in all cases. The characteristic impedances of the first stepped resonators decrease from 68.37 ohms in section 1 to 30.65 ohms in section 7 (impedance ratio: 1.14), whereas for the second resonator the impedance of section 7 is 7.67 ohms (impedance ratio: 1.44).

According to equ.(1) the positions of the steps are about $\lambda_g/4$ and $3\lambda_g/4$ distant from the open aperture; hereby the sensitivity as defined above can be increased compared to the case of the homogeneous resonators with the parameters $Z_{L1}$ and $\beta_1$ and the same resonance mode.

The theoretical results documented in Fig.5 have been calculated by means of an equivalent circuit for the terminating structure considering the total capacitance $C_t$ (Fig.2) and additional capacitances describing the influence of an possible air gap between the inner conductor face and the dielectric material and of the surface roughness of the dielectric material respectively. The capacitance $C_t$ has been calculated /5/ for the different values of the permittivity $\varepsilon_{r1}$ under the assumption that the dielectric material is in close contact to the conductor faces. The influence of the coupling to the feed line has been taken into account by a variation of the electrical line length $l_7$. The additional line sections built by the teflon disks supporting the inner conductor have not yet been considered in the computation.

The results shown in Fig.5 demonstrate that the sensitivity - i.e. the negative slope of the curves shown - can be increased considerably using stepped resonators. Such an increase can be achieved without varying the aperture range, that is without changing the local character of the measurement results. However it is more difficult to position the resonators properly on the dielectric material in this case.

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